

## **Course-7A: Mathematical Special Functions**

(Skill Enhancement Course (Elective), 5 credits)

Course Code:

Max Marks: 100

### **I. Learning Outcomes:**

Students after successful completion of the course will be able to:

1. Understand the Beta and Gamma functions, their properties and relation between these two functions, understand the orthogonal properties of Chebyshev polynomials and recurrence relations.
2. Find power series solutions of ordinary differential equations.
3. solve Hermite equation and write the Hermite Polynomial of order (degree)  $n$ , also find the generating function for Hermite Polynomials, study the orthogonal properties of Hermite Polynomials and recurrence relations.
4. Solve Legendre equation and write the Legendre equation of first kind, also find the generating function for Legendre Polynomials, understand the orthogonal properties of Legendre Polynomials.
5. Solve Bessel equation and write the Bessel equation of first kind of order  $n$ , also find the generating function for Bessel function understand the orthogonal properties of Bessel function.

### **II. Syllabus:** (Hours: Teaching: 75 (incl. unit tests etc. 05), Training: 15)

#### **Unit – 1: Beta and Gamma functions, Chebyshev polynomials** (15h)

1. Euler's Integrals-Beta and Gamma Functions, Elementary properties of Gamma Functions,

Transformation of Gamma Functions.

2. Another form of Beta Function, Relation between Beta and Gamma Functions.
3. Chebyshev polynomials, orthogonal properties of Chebyshev polynomials, recurrence relations, generating functions for Chebyshev polynomials.

#### **Unit – 2: Power series and Power series solutions of ordinary differential equations** (15h)

1. Introduction, summary of useful results, power series, radius of convergence, theorems on Power series
2. Introduction of power series solutions of ordinary differential equation
3. Ordinary and singular points, regular and irregular singular points, power series solution.

**Unit – 3: Hermite polynomials** (15h)

1. Hermite Differential Equations, Solution of Hermite Equation, Hermite polynomials, generating function for Hermite polynomials.
2. Other forms for Hermite Polynomials, Rodrigues formula for Hermite Polynomials, to find first few Hermite Polynomials.
3. Orthogonal properties of Hermite Polynomials, Recurrence formulae for Hermite Polynomials.

**Unit – 4: Legendre polynomials** (15h)

1. Definition, Solution of Legendre's equation, Legendre polynomial of degree n, generating function of Legendre polynomials.
2. Definition of  $P_n(x)$  and  $Q_n(x)$ , General solution of Legendre's Equation (derivations not required) to show that  $P_n(x)$  is the coefficient of  $h^n$ , in the expansion of  $(1 - 2xh + \frac{-1}{h^2})^{\frac{1}{2}}$
3. Orthogonal properties of Legendre's polynomials, Recurrence formulas for Legendre's Polynomials.

**Unit – 5: Bessel's equation** (15h)

1. Definition, Solution of Bessel's equation, Bessel's function of the first kind of order n, Bessel's function of the second kind of order n.
2. Integration of Bessel's equation in series form=0, Definition of  $J_n(x)$ , recurrence formulae for  $J_n(x)$ .
3. Generating function for  $J_n(x)$ , orthogonality of Bessel functions.

**II. Reference Books:**

1. Dr.M.D.Raisinghania, Ordinary and Partial Differential Equations, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.
2. J.N.Sharma and Dr.R.K.Gupta, Differential equations with special functions, Krishna Prakashan Mandir.
3. Shanti Narayan and Dr.P.K.Mittal, Integral Calculus, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.
4. George F.Simmons, Differential Equations with Applications and Historical Notes, Tata McGRAW-Hill Edition, 1994.
5. Shepley L.Ross, Differential equations, Second Edition, John Wiley & sons, New York, 1974.
6. Web resources suggested by the teacher and college librarian including reading material.

#### **IV. Co-Curricular Activities:**

##### **A) Mandatory:**

**1. For Teacher:** Teacher shall train students in the following skills for 15 hours, by taking relevant outside data (Field/Web).

1. Beta and Gamma functions, Chebyshev polynomials.
2. Power series, power series solutions of ordinary differential equations,
3. Procedures of finding series solutions of Hermite equation, Legendre equation and Bessel equation.
4. Procedures of finding generating functions for Hermite polynomials, Legendre Polynomials and Bessel's function.